

HYBRID INFLATION AND SUPERGRAVITY

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Abstract. Hybrid inflation is a natural scenario in the absence of supersymmetry. In the context of supergravity, however, it has to face the naturalness problems of the initial conditions and of the adequate suppression of the inflaton mass. Both can be successfully addressed in a class of models involving Kähler potentials associated with products of $SU(1,1)/U(1)$ Kähler manifolds and “decoupled” fields acquiring large vacuum expectation values through D-terms.

Inflation offers an elegant solution to many cosmological problems [1]. However, “natural” realizations of the inflationary scenario are hard to find. “New” and “chaotic” inflation [1] invoke a very weakly coupled scalar field, the inflaton, in order to reproduce the observed temperature fluctuations $\frac{\Delta T}{T}$ [2] in the cosmic background radiation (CBR). To overcome this naturalness problem Linde proposed the “hybrid” inflationary scenario [3, 4] involving a coupled system of (two) scalar fields which manages to produce the temperature fluctuations in the CBR with natural values of the coupling constants. This is achieved by exploiting the smallness in Planck scale units ($M_P/\sqrt{8\pi} \simeq 2.435515 \times 10^{18} \text{ GeV} = 1$ which are adopted throughout our discussion) of the false vacuum energy density associated with the phase transition leading to the spontaneous breaking of a symmetry in the post-Planck era. In the case that the broken symmetry is a gauge symmetry one of the (two) scalar fields involved is not a gauge singlet.

Linde’s potential is given by

$$V(\varphi, \sigma) = (-\mu^2 + \frac{1}{4}\lambda\varphi^2)^2 + \frac{1}{4}\lambda_1\varphi^2\sigma^2 + \frac{1}{2}\beta\mu^4\sigma^2, \quad (1)$$

where φ, σ are real scalar fields, μ is a mass parameter related to the symmetry breaking scale and $\lambda, \lambda_1, \beta$ are real positive constants. Notice that at $\sigma^2 = \sigma_c^2 = 2\frac{\lambda}{\lambda_1}\mu^2$ the σ -dependent mass-squared of φ , $m_\varphi^2(\sigma) =$

$-\lambda\mu^2 + \frac{1}{2}\lambda_1\sigma^2$, vanishes. Then, for $\sigma^2 > \sigma_c^2$, $m_\varphi^2(\sigma) > 0$ and the potential at fixed σ as a function of φ , namely $V_\sigma(\varphi)$, has a minimum at $\varphi = 0$ with $V_\sigma(0) = \mu^4(1 + \frac{1}{2}\beta\sigma^2)$. For $\sigma^2 < \sigma_c^2$ instead, $m_\varphi^2(\sigma) < 0$ and $V_\sigma(\varphi)$ has a minimum at $|\frac{\varphi}{2}| = \left(-\frac{m_\varphi^2(\sigma)}{\lambda^2}\right)^{\frac{1}{2}}$. Moreover, $|\frac{\varphi}{2}| = \frac{\mu}{\sqrt{\lambda}}, \sigma = 0$ minimizes $V(\varphi, \sigma)$.

Let us assume that $\frac{2}{\beta} \gg \sigma^2 > \sigma_c^2$, $\varphi = 0$ and $\beta \ll 1$. Then, the potential is dominated by the almost constant false vacuum energy density, i.e. $V(0, \sigma) = \mu^4(1 + \frac{1}{2}\beta\sigma^2) \simeq \mu^4$, the “slow-roll” parameters $\epsilon, |\eta| \ll 1$, since $\epsilon \equiv \frac{1}{2} \left(\frac{V'}{V}\right)^2 = \frac{1}{2}\beta^2\sigma^2 \ll \beta$, $\eta \equiv \frac{V''}{V} = \beta$, and the universe experiences an inflationary stage with Hubble parameter $H \simeq \frac{\mu^2}{\sqrt{3}}$. During inflation the motion of the inflaton field σ is governed, in the “slow-roll” approximation, by the equation

$$\frac{d\sigma}{dt} \simeq -\frac{1}{\sqrt{3}}\beta\mu^2\sigma. \quad (2)$$

Inflation ends at $\sigma \simeq \sigma_c$ with a rapid phase transition towards the true minimum $|\frac{\varphi}{2}| = \frac{\mu}{\sqrt{\lambda}}, \sigma = 0$. The number of e-foldings for the cosmic time interval (t_{in}, t_f) , corresponding to a variation of σ between the values σ_{in} and σ_f (with $\sigma_{in}^2 > \sigma_f^2$), is

$$\int_{t_{in}}^{t_f} H dt \simeq \beta^{-1} \ln \frac{\sigma_{in}}{\sigma_f} = N(\sigma_{in}) - N(\sigma_f) \quad (3)$$

with $N(\sigma) \equiv \beta^{-1} \ln \frac{\sigma}{\sigma_c}$. Also the spectral index of density fluctuations $n \simeq 1 + 2\beta$ is almost scale invariant and slightly larger than 1 for $\beta \ll 1$. Assuming, as it turns out to be the case for $\beta \ll 1$, that the measured (quadrupole) anisotropy $\frac{\Delta T}{T} \simeq 6.6 \times 10^{-6}$ is dominated by its scalar component $\left(\frac{\Delta T}{T}\right)_S \simeq \left(12\pi\sqrt{5}V'\right)^{-1} V^{\frac{3}{2}}$ (evaluated at $\sigma = \sigma_H = \sigma_c e^{\beta N_H}$, where $N_H \equiv N(\sigma_H) \simeq 50 - 60$ is the number of e-foldings of “observable” inflation) and choosing $\lambda_1 = \lambda^2$ we have

$$\frac{\mu}{\sqrt{\lambda}} = 12\pi\sqrt{10} \left(\frac{\Delta T}{T}\right)_S \frac{\beta}{\lambda} e^{\beta N_H} \simeq 0.79 \times 10^{-3} \frac{\beta}{\lambda} e^{\beta N_H}. \quad (4)$$

Taking $\beta \sim 10^{-4}$, $\lambda \sim 1$ we obtain an intermediate scale of symmetry breaking $\frac{\mu}{\sqrt{\lambda}} \sim 10^{-7}$ and an electroweak-scale inflaton mass $\sqrt{\beta}\mu^2 \sim 10^{-16}$. Taking, instead, larger values of β we obtain larger scales. For example, $\beta \simeq 1/35$, $\lambda \simeq 10^{-2}$, $N_H \simeq 55$ gives $\frac{\mu}{\sqrt{\lambda}} \simeq 1.1 \times 10^{-2}$ and $\mu \simeq 1.1 \times 10^{-3}$.

At this point we should remark that μ cannot be arbitrarily large since there is an upper bound on the energy density scale $V_{infl}^{\frac{1}{4}} \simeq V_{\sigma_H}^{\frac{1}{4}} \simeq \mu$ where

the “observable” inflation begins. By exploiting the fact that the tensor component $(\frac{\Delta T}{T})_T$ of $\frac{\Delta T}{T}$ satisfies $(\frac{\Delta T}{T})_T^2 \simeq (720\pi^2)^{-1} 6.9V_{\sigma_H} < (\frac{\Delta T}{T})^2$ we immediately derive the bound

$$V_{infl}^{\frac{1}{4}} \simeq V_{\sigma_H}^{\frac{1}{4}} \simeq \mu \lesssim 1.46 \times 10^{-2}. \quad (5)$$

How natural are the initial conditions that lead to the hybrid inflationary scenario [5]? We assume that the energy density ρ of the universe is dominated by $V(\varphi, \sigma)$. Let us start away from the inflationary trajectory and choose the energy density ρ_0 to satisfy the relation $\mu^4 \ll \rho_0 \lesssim 1$. Moreover, we assume that φ^2 starts somewhat below σ^2 . Then, the relevant term in V for our discussion is the term $\frac{1}{4}\lambda^2\varphi^2\sigma^2$. We would like φ to oscillate from the beginning as a massive field due to its coupling to σ and quickly become very close to zero. In contrast σ^2 should stay considerably larger than σ_c^2 . Thus, for $\mu^4 \ll \rho \leq \rho_0 \lesssim 1$ it is required that $\frac{4m_\varphi^2}{9H^2} \gg 1 \gg \frac{4m_\sigma^2}{9H^2}$ or $\varphi^2 \ll \frac{8}{3} \ll \sigma^2$. When $\rho \sim \mu^4$, instead, $|\sigma|$ remains larger than $|\sigma_c|$ provided $\frac{4m_\sigma^2}{3\mu^4} \lesssim 1$ or $\varphi^2 \lesssim \frac{3}{2}\frac{\mu^4}{\lambda^2}$. If we allow $|\sigma_0| \gg 1$, $|\varphi_0|$ does not have to be very small. For example, with the choice $\beta \simeq 1/35$, $\lambda \simeq 10^{-2}$, $\mu \simeq 1.1 \times 10^{-3}$ we could have $|\sigma_0| \simeq 4.5$, $|\varphi_0| \simeq 1$. If, instead, we insist that $|\sigma_0| < 1$ we are forced to start very close to the inflationary trajectory $\rho_0 \simeq \mu^4 \ll 1$ and severely fine tune the starting field configuration ($|\sigma_c| \ll |\sigma_0| < 1$, $|\varphi_0| \lesssim \frac{\mu^2}{\lambda} \ll 1$).

This severe fine tuning becomes more disturbing since the field configuration at the assumed onset of inflation, where $H = H_{infl}$, should be homogeneous over distances $\sim H_{infl}^{-1}$. Notice that H_{infl}^{-1} is larger than the Hubble distance at the end of the Planck era ($\rho = \rho_{in} \simeq 1$) as expanded (according to the expansion law $R \sim \rho^{-\frac{1}{3\gamma}}$, where R is the scale factor of the universe) till the assumed onset of inflation (at $\rho = \rho_{infl}$) by a factor $\frac{H_{infl}^{-1}}{H_{in}^{-1}} \left(\frac{\rho_{infl}}{\rho_{in}} \right)^{\frac{1}{3\gamma}} = \left(\frac{\rho_{infl}}{\rho_{in}} \right)^{-\frac{3\gamma-2}{6\gamma}} \gg 1$, if $\gamma \gtrsim 1$. Therefore, in order for any inflation to start at an energy density scale $\rho_{infl}^{\frac{1}{4}} \simeq V_{infl}^{\frac{1}{4}} \ll 1$, the initial field configuration at $\rho = \rho_{in} \simeq 1$ (where initial conditions should be set) must be very homogeneous over distances $\sim \left(V_{infl}^{-\frac{1}{4}} \right)^{\frac{2}{3\gamma-2}} \gg 1$. Such a homogeneity is hard to understand unless a short period of inflation took place at $\rho \sim 1$ [6] with a number of e-foldings $\gtrsim 2\frac{3\gamma-2}{3\gamma} \ln \left(V_{infl}^{-\frac{1}{4}} \right)$. An early inflationary stage might also eliminate the requirement of severe fine tuning of the field configuration at $\rho = \rho_{in}$ since, in addition to the homogenization of space, it could alter the dynamics during the early stages of the evolution of the universe.

An inflation taking place at an energy density $\rho_1 \gg \rho_{infl}$, however, although eliminates existing inhomogeneities it generates new ones due to quantum fluctuations. These fluctuations are $\sim \frac{H_1}{2\pi}$ for massless fields and generate inhomogeneities over distances $\sim H_1^{-1}$ resulting in a gradient energy density $\sim \frac{H_1^4}{4\pi^2} = \frac{\rho_1^2}{36\pi^2}$ which falls with the expansion only like $R^{-2} \sim \rho^{\frac{2}{3\gamma}}$. The size of this gradient energy density when ρ falls to $\rho_{infl} \simeq V_{infl}$ should be smaller than V_{infl} . This gives an upper bound on the energy density ρ_1 (towards the end) of the first stage of inflation

$$\rho_1 \lesssim (6\pi)^{\frac{3\gamma}{3\gamma-1}} \left(V_{infl}^{\frac{1}{4}} \right)^{2\frac{3\gamma-2}{3\gamma-1}} \quad (\gamma \gtrsim 1) \quad (6)$$

which is somewhat lower than unity and decreases with $V_{infl}^{\frac{1}{4}}$.

Such an early inflationary stage can be easily incorporated into the hybrid model [6]. In particular, if we allow field values considerably larger than unity (e.g. $|\varphi_0| = |\sigma_0| \gtrsim 10$ for $\beta \simeq 1/35$, $\lambda \simeq 10^{-2}$, $\mu \simeq 1.1 \times 10^{-3}$) the original model gives rise to an early chaotic-type inflationary stage at $\rho = \rho_0 \sim \rho_{in}$ which takes care of the initial condition problem.

Linde's potential can be easily obtained in the context of global supersymmetry (SUSY). Let us consider a model with gauge group G which breaks spontaneously at a scale M . The symmetry breaking of G is achieved through a superpotential which includes the terms [7]

$$W = S(-\mu^2 + \lambda\Phi\bar{\Phi}). \quad (7)$$

Here $\Phi, \bar{\Phi}$ is a conjugate pair of left-handed superfields which belong to non-trivial representations of G and break it by their vacuum expectation values (vevs), S is a gauge singlet left-handed superfield, μ is a mass scale related to M and λ a real and positive coupling constant. The superpotential terms in W are the dominant couplings involving the superfields $S, \Phi, \bar{\Phi}$ which are consistent with a continuous R-symmetry under which $W \rightarrow e^{i\vartheta}W$, $S \rightarrow e^{i\vartheta}S$, $\Phi\bar{\Phi} \rightarrow \Phi\bar{\Phi}$. The potential obtained from W is

$$V = |-\mu^2 + \lambda\Phi\bar{\Phi}|^2 + \lambda^2 |S|^2 (|\Phi|^2 + |\bar{\Phi}|^2) + D - terms, \quad (8)$$

where the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. The SUSY minimum $S = 0$, $\Phi\bar{\Phi} = \mu^2/\lambda$, $|\Phi| = |\bar{\Phi}|$ lies on the D-flat direction $\Phi = \bar{\Phi}^*$. By appropriate gauge and R-transformations on this D-flat direction we can bring the complex $\Phi, \bar{\Phi}, S$ fields on the real axis, i.e. $\Phi = \bar{\Phi} \equiv \frac{1}{2}\varphi$, $S \equiv \frac{1}{\sqrt{2}}\sigma$, where φ and σ are real scalar fields. The potential then becomes

$$V(\varphi, \sigma) = (-\mu^2 + \frac{1}{4}\lambda\varphi^2)^2 + \frac{1}{4}\lambda^2\varphi^2\sigma^2. \quad (9)$$

This is Linde's potential (with $\lambda_1 = \lambda^2$) apart from a mass-squared term for σ . A tiny m_σ^2 can be generated as a result of soft SUSY-breaking or a larger one due to the promotion of global SUSY to local as we will see shortly. In the absence of m_σ^2 the necessary slope V' could be provided by radiative corrections [7].

Supersymmetry cannot, of course, remain just global. Thus, we must at some point face the problem of extending the hybrid model to incorporate supergravity. We might naively have thought that we could evade the complications of supergravity by staying at small energies and field values. Unfortunately, there are two reasons for which this is not possible. Firstly, as our earlier discussion made it clear, the problem of initial conditions by definition cannot be addressed at small field values and energies. A second very well-known reason is that, in the case that the potential during inflation is dominated by the F-term, supergravity tends to give a large mass to almost all fields, thereby eliminating most candidate inflatons [4, 8]. This can be easily seen by considering the F-term potential in supergravity

$$V_F = e^K(\cdots), \quad (10)$$

where K is the Kähler potential. Let us assume that our candidate inflaton field S is canonically normalized for $|S|^2 \ll 1$ and the Kähler potential admits an expansion $K = |S|^2 + \dots$. Then,

$$m_S^2 = \frac{\partial^2 K}{\partial S \partial S^*} V_F + \dots = (1 + \dots) V_F + \dots = V_F + \dots \quad (11)$$

Thus, during inflation, no matter how small $V_{infl} \simeq V_{F_{infl}}$ is, there is always a contribution to $m_S^2 \simeq V_{infl}$ or a contribution $\simeq 1$ to the “slow-roll” parameter $|\eta|$. There could very well exist other contributions to η partially cancelling the one just described but their existence will depend on the details of the model. Therefore it seems that in the context of supergravity it is easy to add to the potential of the hybrid model a sizeable mass-squared term for the inflaton σ . We only have to understand why $\beta \equiv \frac{m_\sigma^2}{\mu^4}$ is not of order unity but much smaller.

In order to investigate the effect of supergravity on the simple globally supersymmetric hybrid model discussed above we restrict ourselves to the inflationary trajectory ($\Phi = \bar{\Phi} = 0$) and use the simple superpotential

$$W = -\mu^2 S \quad (12)$$

involving just the gauge singlet superfield S . If the minimal Kähler potential $K = |S|^2$ leading to canonical kinetic terms for σ is employed the “canonical” potential V_{can} acquires a slope and becomes [4, 9, 10]

$$V_{can} = \mu^4 \left(1 - |S|^2 + |S|^4 \right) e^{|S|^2} = \mu^4 \sum_{k=0}^{\infty} \frac{(k-1)^2}{k!} |S|^{2k}. \quad (13)$$

Obviously V_{can} does not allow inflation unless $|S|^2 \ll 1$. From its expansion as a power series in $|S|^2$ we see that, due to an “accidental” cancellation, the linear term in $|S|^2$ is missing and no mass-squared term is generated for σ . Small deviations from the minimal form of the Kähler potential respecting the R-symmetry lead to a Kähler potential [11]

$$K = |S|^2 - \frac{\alpha}{4} |S|^4 + \dots \quad \left(|S|^2 \ll 1 \right). \quad (14)$$

This, in turn, gives rise to a potential admitting an expansion

$$V = \mu^4 \left(1 + \alpha |S|^2 + \dots \right) \quad \left(|S|^2 \ll 1 \right) \quad (15)$$

in which a linear term in $|S|^2$ proportional to the small parameter α is now generated. All higher powers of $|S|^2$ are still present in the series with coefficients only slightly different from the corresponding ones in the expansion of V_{can} .

The above discussion seems to indicate that the only potential source of mass for σ is the next to leading term in the expansion of the Kähler potential in powers of $|S|^2$ which must have a small and negative coefficient. This conclusion is certainly correct if all other fields are assumed to play absolutely no role during inflation. There could exist fields, however, which do not contribute to the superpotential and are G -singlets, but do contribute to the mass-squared of σ if they acquire large vevs. Such fields could destroy the “miraculous” cancellation leading to a massless σ in the case of the minimal Kähler potential [12] but could also generate new “miraculous” cancellations if other types of possibly better motivated Kähler potentials with $\alpha < 0$ are employed [13].

Let us consider a G -singlet chiral superfield Z which does not contribute to the superpotential at all because, for instance, it has non-zero charge, let us say -1 , under an “anomalous” $U(1)$ gauge symmetry and, as we assume, all other superfields which have a $U(1)$ charge can be safely ignored. Also let us assume that $K = K_1(|S|^2) + K_2(|Z|^2)$. Then, with the parameters μ and λ in W renamed as μ' and λ' , the scalar potential (always with $\Phi = \bar{\Phi} = 0$) becomes

$$V = \mu'^4 \left\{ \left| 1 + \frac{\partial K}{\partial S} S \right|^2 \left(\frac{\partial^2 K}{\partial S \partial S^*} \right)^{-1} + \left| \frac{\partial K}{\partial Z} S \right|^2 \left(\frac{\partial^2 K}{\partial Z \partial Z^*} \right)^{-1} - 3 |S|^2 \right\} e^K \\ + \frac{1}{2} g_1^2 \left(\frac{\partial K}{\partial Z} Z - \xi \right)^2, \quad (16)$$

where the first(second) term is the F(D)-term, $\xi > 0$ is a Fayet-Iliopoulos term and g_1 the gauge coupling of the “anomalous” $U(1)$ gauge symmetry.

Minimization of such a potential for fixed $|S|^2$ not much larger than unity, assuming $|S|^2$ takes values away from any points where the potential is singular and $\mu'^2 \ll \xi$, typically gives rise to a $\langle |Z|^2 \rangle \equiv v^2 \sim \xi$ with $\left(\left| \frac{\partial K}{\partial Z} \right|^2 \left(\frac{\partial^2 K}{\partial Z \partial Z^*} \right)^{-1} \right)_{|Z|=v} \sim v^2 \sim \xi$ and a contribution to the mass-squared of σ

$$\delta m_\sigma^2 = \left(\left| \frac{\partial K}{\partial Z} \right|^2 \left(\frac{\partial^2 K}{\partial Z \partial Z^*} \right)^{-1} \right)_{|Z|=v} \mu'^4 e^{K_2(v^2)} \quad (17)$$

of the order of ξ in units of the false vacuum energy density. For the sake of convenience we absorb the factor $e^{K_2(v^2)}$ appearing in the F-term potential in the reintroduced parameters $\mu = \mu' e^{K_2(v^2)/4}$ and $\lambda = \lambda' e^{K_2(v^2)/2}$ obeying the relation $\frac{\mu}{\sqrt{\lambda}} = \frac{\mu'}{\sqrt{\lambda'}}$.

Notice that the contribution of Z to m_σ^2 is positive. Therefore to make use of the above discussion we should find Kähler potentials $K_1(|S|^2)$ whose expansion in powers of $|S|^2$ has a positive next to leading term (i.e. $\alpha < 0$). A class of such Kähler potentials is given by

$$K_1(|S|^2) = -N \ln \left(1 - \frac{|S|^2}{N} \right) \quad (|S|^2 < N), \quad (18)$$

where N is an integer. The corresponding Kähler manifold is the coset space $SU(1,1)/U(1)$ with constant scalar curvature $2/N$. Expanding K_1 in powers of $|S|^2$ we see that $\alpha = -2/N$ and therefore

$$m_\sigma^2 = \left(-\frac{2}{N} + \left| \frac{\partial K}{\partial Z} \right|^2 \left(\frac{\partial^2 K}{\partial Z \partial Z^*} \right)^{-1} \right)_{|Z|=v} \mu^4 \equiv \beta \mu^4. \quad (19)$$

For all N we can make m_σ^2 positive (or, by fine tuning, zero) through appropriately chosen vevs (ξ parameters) of Z -type fields.

It would be very interesting if the contribution of Z to the mass-squared of σ in units of the false vacuum energy density were independent of the value of Z . This is exactly the case if Z enters the Kähler potential through a function K_2 of the “no-scale” type

$$K_2(|Z|^2) = -n \ln \left(-\ln |Z|^2 \right) \quad (0 < |Z|^2 < 1), \quad (20)$$

where n is an integer. The corresponding Kähler manifold is again the coset space $SU(1,1)/U(1)$ with constant scalar curvature $2/n$. Such a choice makes the contribution δm_σ^2 of Z to m_σ^2 an integer multiple of μ^4 , namely $\delta m_\sigma^2 = n\mu^4$. With this choice of K_2 we obtain

$$m_\sigma^2 = \left(-\frac{2}{N} + n \right) \mu^4. \quad (21)$$

Obviously the most interesting cases occur for $N = 1$ or $N = 2$ because $2/N$ is an integer and the option of naturally making m_σ^2 vanish for $n = 2$ or $n = 1$, respectively becomes now available. A small positive m_σ^2 could be subsequently generated through additional Z -type fields which acquire vevs of the order of appropriately chosen ξ parameters.

The choices $N = 1$ or $N = 2$ deserve particular attention for the additional reason that in these cases all supergravity corrections to the F-term potential are proportional to the mass-squared m_σ^2 of the field σ or, equivalently, to the parameter β . This offers the possibility of suppressing or even eliminating all supergravity corrections to the inflationary trajectory by suppressing or making vanish the parameter β . Indeed, substituting the Kähler potential $K_1(|S|^2)$ of Eq. (18) in Eq. (16) and minimizing with respect to Z at fixed $|S|^2$ we obtain for $N = 1, 2$ only

$$V \simeq \mu^4 \left\{ 1 + \beta |S|^2 \left(1 - \frac{|S|^2}{N} \right)^{-N} \right\} \quad (N = 1, 2) \quad (22)$$

(up to terms $\sim \mu^8$) independently of the mechanism chosen to make $\beta \geq 0$. Such models allow for inflation at inflaton field values close to 1 or even slightly larger and lead to a possibly detectable $(\frac{\Delta T}{T})_T$ [13]. For relatively small $|S|^2$ we obtain the original hybrid model. In particular, with the choice of $K_2(|Z|^2)$ of Eq. (20) the combinations $(N, n) = (1, 2)$ and $(N, n) = (2, 1)$ give $\beta = 0$ and consequently a completely flat potential. [As already mentioned a small β could be generated through additional Z -type fields.] These models with $\beta = 0$ could be regarded as a justification for the SUSY hybrid inflationary scenario [7] in which supergravity is neglected completely and the necessary slope V' is provided entirely by radiative corrections.

Let us now discuss the initial conditions in a model with $\beta = 0$ and a classically completely flat inflationary trajectory. Our specific model involves, in addition to the superfields $S, \Phi, \bar{\Phi}$, one G -singlet superfields Z with charge -1 under the “anomalous” $U(1)$ gauge symmetry. The Kähler potential is chosen to be

$$K = -\ln(1 - |S|^2) - 2\ln(-\ln|Z|^2) + |\Phi|^2 + |\bar{\Phi}|^2 \quad (23)$$

($|S|^2 < 1, 0 < |Z|^2 < 1$) with the superpotential always being given by $W = S(-\mu'^2 + \lambda' \Phi \bar{\Phi})$. We define the canonically normalized real scalar fields σ_{infl} and ζ through the relations

$$\tanh \frac{\sigma_{infl}}{\sqrt{2}} \equiv Re S, \quad e^{-\zeta} \equiv -\frac{\xi}{2} \ln |Z|^2, \quad (24)$$

with the complex scalar fields S , Z brought to the real axis by symmetry transformations. To simplify the discussion we further set $\Phi = \bar{\Phi} = \frac{\varphi}{2}$, where φ is a canonically normalized real scalar field, and we consider a truncated version of the complete scalar potential

$$V = \frac{\lambda^2}{4} \varphi^2 \left(\cosh \left(\sqrt{2} \sigma_{infl} \right) - 1 \right) e^{2\zeta} + \frac{1}{2} g_1^2 \xi^2 \left(e^\zeta - 1 \right)^2 \quad (25)$$

(technically justified for $\frac{\mu^2}{\lambda} \ll \varphi^2 \lesssim 10^{-1}$) possessing all its salient features ($\lambda = \lambda' \frac{\xi}{2}$ and $\mu^4 = \mu'^4 \frac{\xi^2}{4}$ such that $\frac{\mu}{\sqrt{\lambda}} = \frac{\mu'}{\sqrt{\lambda'}}$). We assume that initially $|\sigma_{infl_0}| \gg 1$, $e^{\zeta_0} \ll 1$, $\varphi_0^2 \sim 10^{-1}$ and the initial time derivatives of all fields vanish. Notice that $e^{\zeta_0} \ll 1$ is required in order for $\rho_0 \lesssim 1$ if $|\sigma_{infl_0}|$ is sufficiently large. Then, e^ζ starts decreasing further unless the F-term of V is smaller than $\rho_0 e^{\zeta_0}$ to begin with. To ensure a sufficiently fast decrease of φ^2 we assume that $\frac{\partial^2 V}{\partial \varphi^2} \gtrsim \rho$ holds from the beginning which, for the initial conditions adopted, translates into

$$\left(\cosh \left(\sqrt{2} \sigma_{infl_0} \right) - 1 \right) e^{2\zeta_0} \gtrsim \frac{g_1^2 \xi^2}{\lambda^2}. \quad (26)$$

With φ^2 decreasing fast the relation $\frac{1}{V} \frac{\partial V}{\partial \zeta} \simeq \frac{1}{V} \frac{\partial^2 V}{\partial \zeta^2} \simeq -2e^\zeta$ ($e^\zeta \ll 1$) is soon established and the universe experiences a stage of “chaotic” D-term inflation with $H = H_1 \simeq \frac{1}{\sqrt{6}} g_1 \xi (1 - e^\zeta)$ which begins when $\zeta = \zeta_{beg} \lesssim \zeta_0 < 0$. The total number of e-foldings N_{tot} as ζ varies from ζ_{beg} towards its minimum at $\zeta_{min} \simeq 0$ is

$$N_{tot} \gtrsim \frac{1}{2} \left(e^{-\zeta_0} - e \right) \quad (27)$$

(assuming inflation ends at $\zeta_{end} = -1$). Moreover, $\frac{\partial^2 V}{\partial \sigma_{infl}^2} \simeq \varphi^2 \frac{\partial^2 V}{\partial \varphi^2}$. Consequently, even if initially $\frac{\partial^2 V}{\partial \sigma_{infl}^2} \gtrsim \rho$ (i.e. $|\sigma_{infl_0}| \gg 1$), very soon $\frac{\partial^2 V}{\partial \sigma_{infl}^2} \ll \rho$ and $|\sigma_{infl}|$ stays large with φ^2 becoming very small. Thus, when the “chaotic” D-term inflation is over the field configuration is close to the inflationary trajectory but σ_{infl} does not reach its terminal velocity as long as ρ is dominated by the coherent oscillations of the massive field ζ about its minimum. Actually, even if the initial field values violate the condition in Eq. (26) and the field configuration fails to approach the inflationary trajectory during the “chaotic” D-term inflation it may still succeed in approaching it during the period in which ρ is dominated by the oscillating field ζ . The “observable” inflation starts only after $\rho \sim \mu^4$.

A numerical investigation of the complete potential reveals the existence of more natural initial conditions than the above simplified analysis indicates. To provide an example in our model with classically flat inflationary

trajectory we consider the choice $\mu = 2.485 \times 10^{-4}$, $\lambda = 4 \times 10^{-3}$ obtainable [14] in the SUSY hybrid inflation [7]. We also choose $g_1 = \frac{1}{\sqrt{2}}$, $\xi = \frac{1}{\sqrt{12}}$ and $\Phi = \bar{\Phi} = \frac{1}{2}(\varphi + i\psi)$ (along a D-flat direction), where φ , ψ are canonically normalized real scalar fields. Then, it is possible to start at $\rho_0 \simeq 0.0176$ with $\varphi_0 = \psi_0 \leq \sqrt{2}$ (or $\varphi_0 \leq 2$, $\psi_0 = 0$), $\sigma_{infl_0} = 1.7$, $\zeta_0 = -2.5$ and zero initial time derivatives for all fields. An alternative possibility with $\rho_0 \simeq 1$ is to set $\varphi_0 = \psi_0 = 2.2$ (or $\varphi_0 = 3.1$, $\psi_0 = 0$), $\zeta_0 = -2.1$, $\sigma_{infl_0} = 5$, $(\frac{d}{dt}\sigma_{infl})_0 = -1$ and assume that the initial time derivatives for the remaining fields vanish. Thus, our scenario allows for a quite natural starting point involving field values which are neither very small nor very large and an initial energy density $\rho_0 \sim 1$ possibly equally partitioned into kinetic and potential.

In summary, hybrid inflation is a natural scenario in the absence of supersymmetry. In the context of supergravity, however, it has to face two potential problems. These are the suppression of the inflaton mass and the implementation of a mechanism providing reasonable initial conditions. Both problems can be solved in a class of models involving Kähler potentials associated with products of $SU(1,1)/U(1)$ Kähler manifolds and “decoupled” fields acquiring large vevs through D-terms.

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